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On universality of stress-energy tensor correlation functions in supergravity

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Abstract

Using the Minkowski space AdS/CFT prescription we explicitly compute in the low-energy limit the two-point correlation function of the boundary stress-energy tensor in a large class of type IIB supergravity backgrounds with a regular translationally invariant horizon. The relevant set of supergravity backgrounds includes all geometries which can be interpreted via gauge theory/string theory correspondence as being holographically dual to finite temperature gauge theories in Minkowski space-times. The fluctuation-dissipation theorem relates this correlation function computation to the previously established universality of the shear viscosity from supergravity duals, and to the universality of the low energy absorption cross-section for minimally coupled massless scalars into a general spherically symmetric black hole. It further generalizes the latter results for the supergravity black brane geometries with non-spherical horizons.

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1 Introduction

In the framework of gauge theory/string theory correspondence [1] the prescription for the computation of the Lorentzian-signature boundary gauge theory correlation functions was formulated in [2, 3]. This development enabled study of interesting non-equilibrium processes (*e.g.* diffusion and sound propagation) in strongly coupled thermal gauge theories [4–10]. In this paper we explicitly compute boundary stress-energy tensor retarded two-point correlation function in a large class of type IIB supergravity backgrounds with regular translationally invariant horizon ("black branes"). We find that in the low-energy limit this correlation function has a universal dependence on the area of the horizon. The class of relevant supergravity geometries is generic enough to include all geometries holographically dual to finite temperature gauge theories. For the latter subset, the fluctuation-dissipation theorem relates the universal properties of the boundary stress-energy tensor correlation functions to the previously established universality of the shear viscosity in the effective hydrodynamic description of hot gauge theory plasma [11, 12]. Finally, for the black brane geometries that allow for an extension to asymptotically flat space-times[1] the universality of the correlation functions can be related [13, 14] to the universality of low energy absorption cross sections for black holes observed in [15].

In the next section we introduce our conventions and describe type IIB supergravity backgrounds where boundary stress-energy tensor correlators exhibit universal properties. The computation of the correlation functions is delegated to section 3. Appendix contains details of the derivation of the effective bulk action for the graviton fluctuations in supergravity backgrounds of section 2.

2 Description of relevant supergravity backgrounds

Consider static type IIB supergravity backgrounds supported by various fluxes and/or axiodilaton with a regular horizon[2]. We assume that Einstein frame ten-dimensional geometry is a direct warped product of the time direction, $p > 2$ -dimensional Euclidean space R^p, and an arbitrary $q = 9-p$ -dimensional (noncompact) 'transverse space' $\mathcal{M}_q$

[1]This is known to be the true for black brane geometries dual to finite temperature maximally supersymmetric gauge theories. We believe that this is true for all supergravity backgrounds holographically dual to gauge theories.

[2]By a 'regular horizon' we mean singularity-free horizon of finite area.

$$ds^2_{10} \equiv \hat{g}_{MN} d\xi^M d\xi^N = -\Omega_1^2(y)\ dt^2 + \Omega_2^2(y) \left(dx^\alpha dx^\alpha + \tilde{g}(y)_{mn} dy^m dy^n \right) \,, \tag{2.1}$$

where $\alpha = 1 \ldots p$ labels Euclidean directions and $\tilde{g}_{mn}$ is the metric on $\mathcal{M}_q$. The transverse manifold $\mathcal{M}_q$ is assumed to be singularity-free[3] and to have only one $(q-1)$-dimensional boundary component $\partial\mathcal{M}_q$. Also, we assume that curvature invariants of the full ten-dimensional metric are small in Planck units so that the supergravity approximation is valid. It turns out, the key property of the background geometry pertinent to the universality of the correlation functions is the following relation between components of the Ricci tensor of (2.1)

$$R_t^t - R_\alpha^\alpha = 0 \,, \tag{2.2}$$

where there is no summation over α. As emphasized in [12], (2.2) is automatically satisfied for all supergravity geometries holographically dual to strongly coupled finite temperature gauge theories in $R^{1,p}$ Minkowski space-time. The argument goes as follows [12]. First observe that for extremal (zero temperature) backgrounds the Poincaré symmetry of the background geometry ensures that the longitudinal components of the stress tensor can only have the form $T_{\mu\nu} \sim g_{\mu\nu}(\ldots)$. Next note that, while turning on nonextremality involves modifications to the metric as well as to the profile of matter fields over $\mathcal{M}_q$, this has no effect on the structure of $T_{\mu\nu}$. Given explicit expression for the type IIB supergravity matter stress tensor [16], the latter is a trivial consequence of the fact that even off the extremality the axiodilaton and fluxes vary only over $\mathcal{M}_q$, and 3-form fluxes are transverse to $R^{1,p}$ at the extremality. Thus, $T_t^t - T_\alpha^\alpha = 0$ for both extremal and nonextremal backgrounds. The Einstein equation then gives (2.2).

There are interesting non-extremal supergravity geometries which do not honor (2.2). One example is supergravity dual to finite temperature $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills theory with a nonzero chemical potential for a $U(1) \subset SO(6)_R$ R-charge. Kaluza-Klein reduction on the five-sphere of the corresponding supergravity solution is the STU-model [17] of the five-dimensional gauged supergravity. Here (see eqs. (24), (25) of [17])

$$R_t^t - R_\alpha^\alpha = \frac{1}{2} F^2 \,, \tag{2.3}$$

[3] $\mathcal{M}_q$ does not have to be a Calabi-Yau (CY) manifold. In fact, in all explicit examples of gauge/string duality at finite temperature $\mathcal{M}_q$ is not a CY space, as the supersymmetry of the dual gauge theory is broken.

where $F^2 = F_{MN}F^{MN}$ is a square of the field strengths of the five-dimensional $U(1)$ gauge potential corresponding to an R-charge chemical potential. As explained in [18], $F^2 \neq 0$, leading to violation of (2.2). The reason for such a violation is quite simple from the ten-dimensional perspective. Indeed, the 10d uplift of the STU models represents spinning extremal D3 branes [18], which metric involves a cross-term of the form $dtd\phi$ (ϕ is one of the S^5 coordinates), sourced by the 5-form stress tensor $T_{\mu\nu} \not\sim g_{\mu\nu}(\cdots)$. As a result, we expect that in this geometry the boundary stress-energy tensor correlation function will not have a universal form derived in section 3. This in turn implies the deviation from the universal result for the shear viscosity [12]. Needless to say, it will be interesting to explicitly compute this deviation and verify the Kovtun, Son and Starinets (KSS) shear viscosity bound [11].

In what follows we restrict our attention to the geometries satisfying constraint (2.2). Also we take $p = 3$ ($q = 6$). Other cases ($p \neq 3$) can be studied along the same lines, and lead to identical conclusions. We take the following ansatz for type IIB supergravity matter fields, which is compatible (through Einstein equations) with (2.2). Both the axiodilaton $\tau = \tau(y) \equiv C_{(0)} + ie^{-\phi}$ and the 3-form fluxes $G_3 = G_3(y) \equiv F_3 - \tau H_3$ vary[4] only over $\mathcal{M}_6$. Additionally G_3 has nonvanishing components only along $\mathcal{M}_6$. For the 5-form $\mathcal{F}_5$ we assume

$$\mathcal{F}_5 = (1 + \star)[d\omega \wedge dt \wedge dx^1 \wedge dx^2 \wedge dx^3] \,, \tag{2.4}$$

with $\omega = \omega(y)$. Explicit computation of the Ricci tensor components of (2.1) yields [12]

$$\begin{aligned} R_t^t =& \Omega_2^{-2}\Omega_1^{-1}\nabla^2\Omega_1 + 7\Omega_2^{-3}\Omega_1^{-1}\nabla\Omega_1\nabla\Omega_2 \,, \\ R_\alpha^\alpha =& \Omega_2^{-3}\nabla^2\Omega_2 + 6\Omega_2^{-4}\left(\nabla\Omega_2\right)^2 + \Omega_2^{-3}\Omega_1^{-1}\nabla\Omega_1\nabla\Omega_2 \,, \end{aligned} \tag{2.5}$$

where ∇ is with respect to $\tilde{g}_{mn}$. It will be convenient to introduce $\triangle(y)$ as

$$\Omega_1(y) = \Omega_2(y)\triangle(y) \,. \tag{2.6}$$

Given (2.5) we find from (2.2) [12]

$$0 = \nabla^2\triangle + 8\Omega_2^{-1}\nabla\Omega_2\nabla\triangle = \Omega_2^{-8}\nabla\left(\Omega_2^8\nabla\triangle\right) \,. \tag{2.7}$$

We assume that supergravity geometry (2.1) is that of the black brane with a regular Schwarzschild horizon. Horizon of (2.1) is an eight-dimensional submanifold

[4]Here and below we use y to denote collection of coordinates $\{y^m\}$ on $\mathcal{M}_q$.

with a direct product structure $R^3 \times \mathcal{H}_5$, where $\mathcal{H}_5$ is a co-dimension one submanifold of $\mathcal{M}_6$ determined by

$$\triangle\Big|_{\mathcal{H}_5} = 0\,. \tag{2.8}$$

Regularity of the horizon requires that

$$\Omega_2\Big|_{\mathcal{H}_5} \ne 0\,, \qquad \det(h_{mn}) \equiv h\Big|_{\mathcal{H}_5} \ne 0\,, \tag{2.9}$$

where h_{mn} is the induced metric on $\mathcal{H}_5$. Notice that we do not require $\mathcal{H}_5$ to be 'spherical', *i.e.*, depend only on a 'radial' coordinate of $\mathcal{M}_6$. As we approach the boundary $\partial\mathcal{M}_6$ of $\mathcal{M}_6$ we expect the restoration of the four-dimensional Poincaré symmetry. This leads to

$$\triangle\Big|_{\partial\mathcal{M}_6} = 1\,. \tag{2.10}$$

The *zero law* of black hole (brane) thermodynamics requires [19] that the surface gravity (or equivalently the temperature) is constant over the horizon for a stationary black hole. This can be easily understood as a requirement for the absence of conical singularity in the analytically continued Euclidean geometry $t \to t_E = it$. Let n^n be a unit normal vector to $\mathcal{H}_5$ in $\mathcal{M}_6$, *i.e.*,

$$n^n n^m \tilde{g}_{nm}\Big|_{\mathcal{H}_5} = 1\,, \qquad n^n v^m \tilde{g}_{nm}\Big|_{\mathcal{H}_5} = 0\,, \tag{2.11}$$

for any vector $\{v^m\} \in \mathcal{H}_5$. From the definition of $\mathcal{H}_5$ as a horizon (2.8)

$$\nabla\triangle - (n\nabla\triangle)\, n\Big|_{\mathcal{H}_5} = 0\,. \tag{2.12}$$

Moreover, the zero law of black brane thermodynamics implies that

$$n\nabla\triangle\Big|_{\mathcal{H}_5} = \text{const} = 2\pi T\,, \tag{2.13}$$

where T is the Hawking's temperature of the black brane.

We conclude this section with two observations useful for the evaluation of the stress-energy tensor correlation function. First, notice that from (2.12), (2.13)

$$(\nabla\triangle)^2\Big|_{\mathcal{H}_5} = \text{const} = (2\pi T)^2\ . \tag{2.14}$$

Second, the area $\mathcal{A}_8$ of the black brane horizon is

$$\mathcal{A}_8 = V_3 \int_{\mathcal{H}_5} d^5\xi \sqrt{h}\ \Omega_2^8\,, \tag{2.15}$$

where V_3 is (a divergent) area of R^3.

3 Boundary stress-energy tensor two-point correlation functions

In this section, using prescription [2], we compute retarded Green's function of the boundary stress-energy tensor $T_{\mu\nu}(t,x^\alpha)$ ($\mu=\{t,x^\alpha\}$) at zero spatial momentum, and in the low-energy limit $\omega\to 0$,

$$G^R_{12,12}(\omega,0)=-i\int dtd^3x\ e^{i\omega t}\theta(t)\langle[T_{12}(t,x^\alpha),T_{12}(0,0)]\rangle\,. \tag{3.1}$$

We find

$$G^R_{12,12}(\omega,0)=-\frac{i\omega s}{4\pi}\left(1+\mathcal{O}\left(\frac{\omega}{T}\right)\right)\,, \tag{3.2}$$

where

$$s=\frac{\mathcal{A}_8}{4V_3G_N} \tag{3.3}$$

is the Bekenstein-Hawking entropy density of the black brane. $G_N=\frac{k_{10}^2}{8\pi}$ is a ten-dimensional Newton constant. Given (3.2) and the Kubo relation

$$\eta=\lim_{\omega\to 0}\frac{1}{2\omega i}\left[G^A_{12,12}(\omega,0)-G^R_{12,12}(\omega,0)\right]\,, \tag{3.4}$$

where $G^A(\omega,0)=\left(G^R(\omega,0)\right)^\star$ is advanced Green's function, we can reproduce the universality of shear viscosity η of strongly coupled gauge theories from supergravity [12]

$$\frac{\eta}{s}=\frac{1}{4\pi}\,. \tag{3.5}$$

We begin computation of (3.1) recalling that the coupling between the boundary value of the graviton and the stress-energy tensor of a gauge theory is given by $\delta g_2^1T_1^2/2$. According to the gauge/gravity prescription, in order to compute the retarded thermal two-point function (3.1) we should add a small bulk perturbation $\delta g_{12}(t,y)$ to the metric (2.1), and compute the on-shell action as a functional of its boundary value $\delta g_{12}^b(t)$. Simple symmetry arguments [5] show that for a perturbation of this type and metric of the form (2.1) all other components of a generic perturbation $\delta g_{\mu\nu}$ can be consistently set to zero. It will be convenient to introduce a field $\varphi=\varphi(t,y)$,

$$\varphi=\frac{1}{2}g^{\alpha\alpha}\ \delta g_{12}=\frac{1}{2}\Omega_2^{-2}\ \delta g_{12}\,. \tag{3.6}$$

Following [2, 3], retarded correlation function $G^R_{12,12}(\omega,0)$ can be extracted from the (quadratic) boundary effective action $S_{boundary}$ for the metric fluctuations φ^b,

$$\varphi^b(\omega)=\int\frac{d^4k}{(2\pi)^4}e^{-i\omega t}\ \varphi(t,y)\Big|_{\partial\mathcal{M}_6}\,, \tag{3.7}$$

given by

$$S_{boundary}[\varphi^b] = \int \frac{d^4k}{(2\pi)^4}\ \varphi^b(-\omega)\ \mathcal{F}(\omega, y)\ \varphi^b(\omega)\Big|_{\mathcal{H}_5}^{\partial\mathcal{M}_6}\,, \tag{3.8}$$

as

$$G^R_{12,12}(\omega, 0) = \lim_{\partial\mathcal{M}_6^r \to \partial\mathcal{M}_6}\ 2\ \mathcal{F}^r(\omega, y)\,. \tag{3.9}$$

The boundary metric functional is defined as

$$S_{boundary}[\varphi^b] = \lim_{\partial\mathcal{M}_6^r \to \partial\mathcal{M}_6} \left(S^r_{bulk}[\varphi] + S_{GH}[\varphi] + S^{counter}[\varphi] \right)\,, \tag{3.10}$$

where S^r_{bulk} is the bulk Minkowski-space type IIB supergravity action on a cut-off space: $\mathcal{M}_6$ in (2.1) is regularized by a compact manifold $\mathcal{M}_6^r$ with a boundary $\partial\mathcal{M}_6^r$. Also, S_{GH} is the standard Gibbons-Hawking term over the regularized boundary $\partial\mathcal{M}_6^r$. The regularized bulk action S^r_{bulk} is evaluated on-shell for the bulk metric fluctuations $\varphi(t, y)$ subject to the following boundary conditions:

$$\begin{aligned} &(a):\ \lim_{\partial\mathcal{M}_6^r \to \partial\mathcal{M}} \varphi(t, y) = \varphi^b(t)\,, \\ &(b):\ \varphi(t, y) \text{ is an incoming wave at the horizon } \mathcal{H}_5\,. \end{aligned} \tag{3.11}$$

The purpose of the boundary counterterm $S^{counter}$ is to remove divergent (as $\partial\mathcal{M}_6^r \to \partial\mathcal{M}_6$) and ω-independent contributions from the kernel $\mathcal{F}$ of (3.8).

Effective bulk action for $\varphi(t, y)$ in supergravity backgrounds specified in previous section (derivation details are given in Appendix) takes the following form

$$\begin{aligned} S_{bulk}[\varphi] = &\frac{1}{2k_{10}^2} \int d^{10}\xi\ \Omega_1 \Omega_2^9 \sqrt{\tilde{g}} \Big[\Omega_1^{-2} \left\{ \tfrac{1}{2} \left(\partial_t \varphi\right)^2 - \partial_t^2 \left(\varphi^2\right) \right\} \\ &+ \Omega_2^{-2} \left\{ -\tfrac{1}{2} \left(\nabla\varphi\right)^2 + \nabla^2 \left(\varphi^2\right) + \nabla(\ln\Omega_1) \nabla \left(\varphi^2\right) + 8 \nabla(\ln\Omega_2) \nabla \left(\varphi^2\right) \right\} \\ &+ \varphi^2 \Omega_1^{-1} \Omega_2^{-9} \nabla \left(\Omega_2^7 \nabla\Omega_1\right) \Big]\,, \end{aligned} \tag{3.12}$$

or equivalently

$$\begin{aligned} S_{bulk}[\varphi] \equiv \int d^{10}\xi\ \mathcal{L}_{10} = &\frac{1}{2k_{10}^2} \int d^{10}\xi\ \Big[\\ &\Omega_1 \Omega_2^9 \sqrt{\tilde{g}} \left\{ \frac{\Omega_1^{-2}}{2} \left(\partial_t\varphi\right)^2 - \frac{\Omega_2^{-2}}{2} \left(\nabla\varphi\right)^2 \right\} \\ &+ \left\{ -\partial_t \left(\sqrt{\tilde{g}} \Omega_1^{-1} \Omega_2^9\ \partial_t(\varphi^2) \right) + \nabla \left(\sqrt{\tilde{g}} \Omega_1 \Omega_2^7\ \nabla(\varphi^2) \right) + \nabla \left(\sqrt{\tilde{g}} \varphi^2 \Omega_1 \Omega_2^6\ \nabla(\Omega_2) \right) \right\} \\ &+ \varphi^2 \sqrt{\tilde{g}} \left\{ \nabla \left(\Omega_2^7 \nabla\Omega_1\right) - \nabla \left(\Omega_1 \Omega_2^6 \nabla\Omega_2\right) \right\} \Big]\,. \end{aligned} \tag{3.13}$$

The second line in (3.13) is the effective action for minimally coupled scalar in geometry (2.1), the third line is a total derivative. Finally, given (2.7), the last line in (3.13) identically vanishes. Thus, bulk equation of motion for φ is that of a minimally coupled scalar in (2.1). Decomposing φ as

$$\varphi(t,y)=e^{-i\omega t}\varphi_\omega(y)\,, \tag{3.14}$$

we find

$$\nabla\left(\Omega_1\Omega_2^7\nabla\varphi_\omega\right)+\omega^2\Omega_1^{-1}\Omega_2^9\varphi_\omega=0\,. \tag{3.15}$$

Similar to [9], a low-frequency solution of (3.15) which is an incoming wave at the horizon, and which near the boundary satisfies

$$\lim_{\partial\mathcal{M}_6^r\to\partial\mathcal{M}_6}\ \varphi_\omega(y)=1\,, \tag{3.16}$$

can be written as

$$\varphi_\omega(y)=\triangle^{-i\omega Q}\ \biggl(F_0+i\omega Q\ F_\omega+\mathcal{O}(\omega^2)\biggr)\,, \tag{3.17}$$

where $\triangle$ is defined as in (2.6) and $F_0=F_0(y)$, $F_\omega=F_\omega(y)$. Exponent $Q>0$ determines a leading singularity of φ_w at the horizon. We find

$$Q=\frac{1}{\sqrt{(\nabla\triangle)^2}}\bigg|_{\mathcal{H}_5}=\frac{1}{2\pi T}\,, \tag{3.18}$$

where we used (2.14). Notice that the fact that Q is constant is related to the zero law of the black brane thermodynamics! Smooth at the horizon functions $\{F_0,\ F_\omega\}$ satisfy following partial differential equations :

$$\begin{aligned}
0=&\nabla\left(\triangle\Omega_2^8\nabla F_0\right)\,,\\
0=&\nabla\left(\triangle\Omega_2^8\nabla F_w\right)-2\Omega_2^8\nabla\triangle\nabla F_0\,,
\end{aligned} \tag{3.19}$$

with the general solution (recall (2.7))

$$\begin{aligned}
F_0&=c_0+c_1\ \ln\triangle\,,\\
F_\omega&=c_1\ (\ln\triangle)^2+c_2\ \ln\triangle+c_3\,,
\end{aligned} \tag{3.20}$$

where c_i are integration constants. The only solution (3.20) nonsingular at the horizon which also satisfies (3.16) is

$$F_0(y)=1\,,\qquad F_\omega(y)=0\,. \tag{3.21}$$

Thus,

$$\varphi(t,y) = e^{-i\omega t}\ \triangle^{-i\omega Q}\ \left(1+\mathcal{O}(\omega^2)\right)\,. \tag{3.22}$$

Once the bulk fluctuations are on-shell (*i.e.*, satisfy equations of motion) the bulk gravitational Lagrangian becomes a total derivative. From (3.13) we find (without dropping any terms)

$$2k_{10}^2\ \mathcal{L}_{10} = \partial_t J^t + \nabla J^y \equiv \partial_t J^t + \nabla_m\left(\tilde{g}^{mn} J^y_n\right)\,, \tag{3.23}$$

where

$$\begin{aligned} J^t =& -\frac{3}{2}\sqrt{\tilde{g}}\Omega_1^{-1}\Omega_2^9\ \varphi\partial_t\varphi\,,\\ J^y =& \frac{3}{2}\sqrt{\tilde{g}}\Omega_1\Omega_2^7\ \varphi\nabla\varphi + \sqrt{\tilde{g}}\Omega_1\Omega_2^6\nabla\Omega_2\ \varphi^2\,. \end{aligned} \tag{3.24}$$

Additionally, the Gibbons-Hawking term provides an extra contribution so that

$$J^y \to J^y - 2\sqrt{\tilde{g}}\Omega_1\Omega_2^7\ \varphi\nabla\varphi\,. \tag{3.25}$$

We are now ready to extract the kernel $\mathcal{F}$ of (3.8). The regularized boundary effective action for φ is

$$\begin{aligned} S_{boundary}[\varphi]^r =& S^r_{bulk}[\varphi] + S_{GH}[\varphi] + S^{counter}[\varphi]\\ =& \frac{1}{2k_{10}^2}\int dt d^3x\ \int_{\mathcal{M}_6^r} d^6y\ \left(\partial_t J^t + \nabla J^y\right) + \int dt d^3x\ \int_{\partial\mathcal{M}_6^r} d^5\xi\ \mathcal{L}^{counter}[\varphi]\\ =& \frac{1}{2k_{10}^2}\int dt d^3x\ \int_{\partial\mathcal{M}_6^r} d^5\xi\ \sqrt{h}\biggl(-\frac{1}{2}\Omega_1\Omega_2^7\ N^n\partial_n\varphi\ \varphi\\ &+\Omega_1\Omega_2^6\ N^n\partial_n\Omega_2\ \varphi^2 + \frac{2k_{10}^2}{\sqrt{h}}\ \mathcal{L}^{counter}[\varphi]\biggr)\,, \end{aligned} \tag{3.26}$$

where we used Stoke's theorem and, as prescribed in [2], maintained only the boundary $\partial\mathcal{M}_6^r$ contribution, N^n is a unit outward normal to $\partial\mathcal{M}_6^r$ and $h = \det(h_{ij})$ is a determinant of the induced metric on $\partial\mathcal{M}_6^r$. The counter-term lagrangian $\mathcal{L}^{counter}$ should be constructed in such a way as to remove any divergent and ω-independent contributions from the effective boundary action in the limit $\partial\mathcal{M}_6^r \to \partial\mathcal{M}_6$. Given (3.22), it is easy to see that the latter is achieved with

$$\mathcal{L}^{counter}[\varphi] = -\frac{1}{2k_{10}^2}\sqrt{h}\Omega_1\Omega_2^6\ N^n\partial_n\Omega_2\ \varphi^2\,. \tag{3.27}$$

It is a very interesting problem to represent an appropriate counter-term as a local functional of boundary metric and matter fields invariants. Though many impressive results in this direction are obtained for large classes of specific supergravity backgrounds (see [20] and references therein), local counterterm expressions for supergravity backgrounds as generic as we are discussing here are not known. Clearly, the strong form of the gauge theory/string theory correspondence (in the context of four-dimensional renormalizable gauge theories with or without supersymmetry) implies that such local representation must exist. As we demonstrate shortly, counter-term lagrangian (3.27) leads to finite correlation functions of the renormalized boundary action. Thus, even though we don't know the divergent structure of the regularized boundary action (3.26), (3.27) must remove all present divergences of the latter. Substituting (3.22) into (3.26) we can obtain $\mathcal{F}^r(\omega, y)$

$$
\begin{aligned}
\mathcal{F}^r(\omega, y) &= -\frac{i\omega Q}{4k_{10}^2}\left(1+\mathcal{O}\left(\frac{\omega}{T}\right)\right)\int_{\partial\mathcal{M}_6^r} d^5\xi\sqrt{h}\ \triangle^{-1}\Omega_1\Omega_2^7\ N^n\partial_n\triangle \\
&= -\frac{i\omega Q}{4k_{10}^2}\left(1+\mathcal{O}\left(\frac{\omega}{T}\right)\right)\int_{\partial\mathcal{M}_6^r} d^5\xi\sqrt{h}\ \Omega_2^8\ N^n\partial_n\triangle\,,
\end{aligned}
\tag{3.28}
$$

where we recalled the definition of $\triangle$. Eq. (2.7) implies that

$$
0 = \int_{\mathcal{M}_6^r} d^6y\sqrt{\tilde{g}}\ \nabla\left(\Omega_2^8\nabla\triangle\right)\,. \tag{3.29}
$$

Application of Stoke's theorem to (3.29) leads to[5]

$$
\begin{aligned}
\int_{\partial\mathcal{M}_6^r} d^5\xi\sqrt{h}\ \Omega_2^8\ N^n\partial_n\triangle &= \int_{\mathcal{H}_5} d^5\xi\sqrt{h}\ \Omega_2^8\ n^n\partial_n\triangle \\
&= \int_{\mathcal{H}_5} d^5\xi\sqrt{h}\ \Omega_2^8\ (2\pi T) = 2\pi T\ \frac{\mathcal{A}_8}{V_3}\,,
\end{aligned}
\tag{3.30}
$$

where in the second line we used (2.13), (2.15). Thus,

$$
\begin{aligned}
\mathcal{F}(\omega, y) &= \lim_{\partial\mathcal{M}_6^r\to\partial\mathcal{M}_6}\mathcal{F}^r(\omega, y) = -\frac{i\omega Q(2\pi T)\mathcal{A}_8}{4V_3k_{10}^2} \\
&= -\frac{i\omega\mathcal{A}_8}{4V_3k_{10}^2} = -\frac{i\omega s}{8\pi}\,,
\end{aligned}
\tag{3.31}
$$

where we used (3.18), (3.3). From (3.9) we obtain quoted result (3.2).

[5]The n^m normal vector is pointing inward, hence the sign.

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Appendix

Here we discuss effective action for the perturbation of metric (2.1)

$$ds_{10}^2 \to ds_{10}^2 + \delta g_{12}(t,y)\ dx^1 dx^2 \equiv ds_{10}^2 + 2\varphi(t,y)\Omega_2^2(y)\ dx^1 dx^2\,, \tag{3.32}$$

in a class of supergravity backgrounds specified in section 2. When applicable, we use notations and results of [21].

Type IIB supergravity action reads

$$\begin{split} S_{IIB} =& \frac{1}{2k_{10}^2}\int d^{10}\xi\ \sqrt{-\hat{g}}\biggl\{R_{10} - \frac{\partial_M\tau\partial^M\bar{\tau}}{2(\mathrm{Im}\,\tau)^2} - \frac{G\cdot\overline{C}}{12} - \frac{F_{(5)}^2}{4\cdot 5!}\biggr\} \\ &+ \frac{1}{8ik_{10}^2}\int\ C_{(4)}\wedge G\wedge\overline{G}\,, \end{split} \tag{3.33}$$

where $C_{(4)} = 4\omega\ dt\wedge dx^1\wedge dx^2\wedge dx^3$, $F_{(5)} = 4\mathcal{F}_5$, also

$$\begin{split} &\tau = i\frac{1+\mathcal{B}}{1-\mathcal{B}}\,, \qquad f = \frac{1}{(1-\mathcal{B}\mathcal{B}^\star)^{1/2}}\,, \\ &G = f(1-\mathcal{B})G_3\,. \end{split} \tag{3.34}$$

Above redefinition is convenient to utilize results of [21].

We would like to evaluate (3.33) in the deformed metric (3.32) to quadratic order in φ. We find

$$\sqrt{-\hat{g}} \to \sqrt{-\hat{g}}\left(1 - \frac{1}{2}\varphi^2\right)\,, \tag{3.35}$$

$$R_{10} \to R_{10} + R_{10}^{(\varphi^2)}\,, \tag{3.36}$$

where

$$\begin{split} R_{10}^{(\varphi^2)} =& \Omega_1^{-2}\left\{\tfrac{1}{2}\left(\partial_t\varphi\right)^2 - \partial_t^2\left(\varphi^2\right)\right\} \\ &+ \Omega_2^{-2}\left\{-\tfrac{1}{2}\left(\nabla\varphi\right)^2 + \nabla^2\left(\varphi^2\right) + \nabla(\ln\Omega_1)\nabla\left(\varphi^2\right) + 8\nabla(\ln\Omega_2)\nabla\left(\varphi^2\right)\right\}\,. \end{split} \tag{3.37}$$

Since the axiodilaton vary only over $\mathcal{M}_6$, its bulk action contribution is not affected by the perturbation (3.32)

$$-\frac{\partial_M \tau \partial^M \bar{\tau}}{2(\operatorname{Im}\tau)^2} \to -\frac{\partial_M \tau \partial^M \bar{\tau}}{2(\operatorname{Im}\tau)^2} = -T_M^{(1)M}\,, \tag{3.38}$$

where $T_{MN}^{(1)}$ is the energy momentum tensor of the axiodilaton (see eq. (3.10) of [21]). Similarly we have

$$-\frac{1}{12} G \cdot \overline{G} \to -\frac{1}{12} G \cdot \overline{G} = -2T_M^{(3)M}\,, \tag{3.39}$$

where $T_{MN}^{(3)}$ is the energy momentum tensor of three index antisymmetric tensor field (see eq. (3.11) of [21]). One has to be careful with evaluation of the action of the self-dual 5-form. A correct prescription to do this was explained in [22]. Thus [21],

$$-\frac{F_{(5)}^2}{4\cdot 5!} \to -\frac{F_{(5)}^2}{4\cdot 5!}\left(1+\varphi^2\right) = 8\Omega_1^{-2}\Omega_2^{-8}\ \left(\nabla\omega\right)^2\left(1+\varphi^2\right)\,, \tag{3.40}$$

also [21]

$$\begin{aligned}
&\frac{1}{8ik_{10}^2}\int\ C_{(4)}\wedge G\wedge\overline{G} \to \frac{1}{8ik_{10}^2}\int\ C_{(4)}\wedge G\wedge\overline{G} \\
&= \frac{1}{8ik_{10}^2}\int\ 8\omega\ dt\wedge dx^1\wedge dx^2\wedge dx^3\wedge G\wedge\bar{G} \quad I.\ Papadimitriou and K.\ Skenderis, \text{“}Correlation func \\
&= \frac{1}{2k_{10}^2}\int\ dtd^3x\int_{\mathcal{M}_6} d^6y\sqrt{\tilde{g}}\left(\frac{i\omega}{3}G\cdot\star_6\bar{G}\right).
\end{aligned}$$

Collecting (3.35)-(3.41) we find

$$S_{IIB} \to S_{IIB} + S_{bulk}[\varphi]\,, \tag{3.42}$$

where

$$S_{bulk}[\varphi] = \int d^{10}\xi\Omega_1\Omega_2^9\sqrt{\tilde{g}}\left(R_{10}^{(\varphi^2)} - \frac{1}{2}\varphi^2\left[R_{10} - T_M^{(1)M} - 2T_M^{(3)M} + \frac{F_{(5)}^2}{4\cdot 5!}\right]\right)\,. \tag{3.43}$$

The trace of Einstein equations implies (the self-dual 5-form does not contribute)

$$R_{10} = T_M^{(1)M} + T_M^{(3)M}\,. \tag{3.44}$$

Additionally, the tt-component of Einstein equations is [21]

$$\begin{aligned}
R_{tt} =& \frac{1}{2}\ \Omega_1^2 T_M^{(3)M} + 4\Omega_2^{-8}\left(\nabla\omega\right)^2 \\
=& \frac{1}{2}\ \Omega_1^2\left(T_M^{(3)M} - \frac{F_{(5)}^2}{4\cdot 5!}\right).
\end{aligned} \tag{3.45}$$

So we can rewrite (3.43) as

$$S_{bulk}[\varphi] = \int d^{10}\xi \Omega_1 \Omega_2^9 \sqrt{\tilde{g}} \left(R_{10}^{(\varphi^2)} + \varphi^2 \Omega_1^{-2} R_{tt} \right) . \tag{3.46}$$

Since

$$R_{tt} = \Omega_1 \Omega_2^{-9} \, \nabla \left(\Omega_2^7 \nabla \Omega_1 \right) , \tag{3.47}$$

we identify (3.46) with (3.12).

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